

Adding Strategies

The answer in each empty box should be the *sum* of the number at the top of the column and the left side of the row. A *sum* is the add-together answer. In other words, where the 1 column meets the 1 row, write 2. When each strategy becomes easy for you, move on (could be a minute or a week! Everyone learns each strategy differently).

- Adding Zero. Go ahead and fill in all the sums in the zero column and in the zero row. Piece of cake, since adding zero to a number doesn't change its value! I always like to think of numbers as numbers of cookies. If I have four cookies in my basket and Alice comes and adds zero cookies to my basket, I still have four cookies. Lightly color in the boxes which you've filled in.

- Counting On. Adding 1 or 2 or 3 is just like counting 1 or 2 or 3 more! When I'm adding 2 to 12, for example, I just sort of count under my breath "12 ... 13, 14." Go ahead and put answers in all the rows and columns for adding 1 or 2 or 3, counting to yourself, then using a new color to lightly fill in those boxes.

- Doubles. There's a special group of sums which are perfect doubles: $4+4$, $5+5$, $6+6$. Work out these sums, color them in with a new color, and you'll find a pattern on your grid. It's OK to use this color for the doubles of 0, 1, 2, and 3 to make the pattern complete!

- Almost Doubles. OK, now I know that $4+4=8$. Great! There are sums which are very close to doubles, like $4+5$. Obviously that's only one more than $4+4$, so the sum is one more than 8... 9! See which of the *almost* doubles you can figure out and color.

- Adding Ten. Just try it. Find the missing sums in the 10 row and the 10 column. See what the tens-digit did? So easy! Works every time! Once you've learned those, do the same for adding *almost* 10; in other words, answer the sums in the 9 row and column, then the 11 row and column.

- Making Ten. When you can add things up and make ten, it's *very* useful. Find them, fill in the sum 10, and color them in a way that stands out. It's OK to color over previously answered boxes to make the pattern show up.

- Leftovers. By now, not too many boxes remain unanswered. Find the sums for them and leave them white. These are the ones you simply memorize.

When you have your own big paper, make yourself a grid *larger* than 12 by 12 and fill in all those sums!

Multiplication Strategies

When you use this grid for practicing your multiplication tables, you are learning a powerful tool which you'll use for the rest of your life! You're also training your brain to be agile in a certain way, a skill that no calculator can give you, and which is most easily learned as a kid. The answer to multiplication is called the *product*. For those of us who like math trivia, the number you begin with is called the *multiplicand* and the number you multiply by is the *multiplier*. For those of us who like baked goods, the multiplicand is how many cookies in each basket and the multiplier is how many baskets.

Multiplication can also answer the question "how many tiles do I need to fill in the rectangle that is *some number* tiles wide by *some other number* tiles long." It seems that the ancient Greeks spent a great deal of time making and then staring at their tiled mosaic floors, so a great deal of multiplication and geometry is based on having a tiled floor.

- Multiply by Zero. The product is zero, every time. Even if five friends bring baskets, if there are zero cookies in each basket, you're doomed, cookie-wise. Also, if every basket has a dozen cookies but zero friends remembered to bring their baskets... no cookies. Very sad. Fill in those blanks and color them.

- Multiply by One. Ignore 1×0 and 0×1 , those products are claimed forever by the multiplying by zero rule. For everyone else, multiplying by one gets you the *other* number you multiplied by. Each basket has one cookie, five friends brought their baskets, that's five cookies. All good.

- Doubling. Great strategy. Doubling is "Multiplying by two". I want you to fill in the 2 row and the 2 column and color in your new products. *Now go forth into the world doubling every number you see!* Doubling is both an excellent exercise for your brain and a phenomenally useful tool! Do this in the car every time you pass a sign with a number on it.

- Multiply by Ten. This is one powerful strategy. Look at that 10. It's a 1 in the tens place and a 0 in the ones place, right? Just hold that 0 off to the side in your head for a moment. Write down the answer as though it were a "multiply by one" problem. Now take that zero you were holding and write it to the right in the ones place where it came from in the first place. As my daughter says, "Easy-peasy-lemon-squeezy". You will do the same thing someday with 100 and 1,000 and so on, just remember the right number of zeroes to plonk down into their original places.

- Multiply by Five. Ever counted by fives? There's a definite rhythm to it, and your musical memory will keep the five tables for you forever. You can probably extend the products way beyond 5×12 just by following the rhythm pattern.

- Perfect squares. Aah. Perfect squares. That means the multiplicand and the multiplier are the *same number*. Like 3×3 or 7×7 . You need to find out those by figuring or calculating or whatever, but do you see the pattern they make on the grid? Color them. Even if they're already another color, color them. These are the backbone of the multiplication table. Do you see that each one makes an *actual* square? Put your finger on the product of 5×5 . Trace a line from that product up to the dashed line, leftward to the other dashed line, down to the 5 row and back to the product. You just traced a perfect square. In fact, the number of little tiles in that square is 25, the product of 5×5 . There is so much you're going to do with perfect squares in your life and it's *all* totally cool. *Now go memorize those perfect squares! Yes, I mean memorize!* When you feel ready, I want you to figure out and memorize the perfect squares up to 20×20 . Absolutely mind-bogglingly useful. Challenge yourself by memorizing in the other direction! "If I have 64, what is that the square of?" Hint: the answer "what is that the square of" is called the *square root*, and it's a powerful tool as well.

Fraction Strategies

Once you're very familiar with fractions, you may have noticed something. Life is no longer about cookies, its about pizza.

Another thing is that you have lots of opportunities to re-name fractions. Simplifying them, making them more complex, making them match other fractions - all of these are easier with your multiplication grid.

- Complicating Fractions. I mean, let's say I get $1/3$ of the pizza, but it's not cut into thirds. What are the more complex fractions that mean the same as $1/3$? Put your fingers on the 1 and the 3 from the line below the dashed line (not from the top because we don't want to get distracted by anything from the zero line).

Slide your fingers down one square. Are you resting on 2 and 6? There's your first answer, $2/6$ is the same as $1/3$. Slide down one more square. 3 and 9 mean that $3/9$ is the same as $1/3$. As long as your fingers stay on the same row, you'll be resting on numbers that tell you an equivalent fraction.

This works for *any* fraction in which the simple form is made from numbers between 1 and 12! Go ahead, try it - try $3/11$ ths!

- Simplifying Fractions. Well, the reverse operation is easy. If you have a large-number looking fraction, like $36/48$, and you need to simplify it, just scan along each line (a ruler might help you see only one line at a time). You're looking for a line which has both 36 and 48 on it. Did you see them on the 6 line? One finger on each and run to the top of those columns... 6 and 8! A simpler name for $36/48$ is $6/8$. Sweet!

You're not quite done. Now you need to check to see if 6 and 8 are together on any lines. Check the 2 line. Run up from the 6 and the 8 on that line - you'll find that an even simpler fraction of $3/4$ is equivalent to $6/8$ and $36/48$. Once you've checked in this way a couple of times, you can be sure you've found the *simplest* form of the fraction. You always want to go down to the simplest form.

This won't work forever and ever with such a small grid, but it will do for your first year with fractions.

- Greatest Common Factor. When you simplify those fractions, it is often useful to remember which lines you found your numbers on. In the example above, you found your numbers on the 6 line *and then* you found the new numbers you were working with on the 2 line. Multiply those together to get 12. 12 is called the *greatest common factor* of 36 and 48. A factor is either the multiplicand or the multiplier; as you've found by now, the two can be interchangeable. You may find the greatest common factor useful if you're working a more complex math problem. You will

definitely find it useful if you're using your math to solve a real-life problem, like measuring and cutting the pieces to build a cabinet.

- Lowest Common Denominator. To make the denominator (the bottom part) of any fraction match the one from another fraction, use your multiplication grid. Let's say we're interested in adding $1/6$ and $1/4$. You can't add sixths and fourths, they're different things, so you need to find the denominator with which *both* fractions could be renamed. Look in your 6 column and your 4 column. Find the *lowest* number which is in both columns. Did you find 12 to be your answer? Good. You're going to re-name each fraction to 12ths, using your "Complicating Fractions" skills.

Addition Grid

	00	01	02	03	04	05	06	07	08	09	10	11	12
00	00	01	02	03	04	05	06	07	08	09	10	11	12
01	01	02	03	04	05	06	07	08	09	10	11	12	13
02	02	03	04	05	06	07	08	09	10	11	12	13	14
03	03	04	05	06	07	08	09	10	11	12	13	14	15
04	04	05	06	07	08	09	10	11	12	13	14	15	16
05	05	06	07	08	09	10	11	12	13	14	15	16	17
06	06	07	08	09	10	11	12	13	14	15	16	17	18
07	07	08	09	10	11	12	13	14	15	16	17	18	19
08	08	09	10	11	12	13	14	15	16	17	18	19	20
09	09	10	11	12	13	15	15	16	17	18	19	20	21
10	10	11	12	13	14	16	16	17	18	19	20	21	22
11	11	12	13	14	15	17	17	18	19	20	21	22	23
12	12	13	14	15	16	18	18	19	20	21	22	23	24

	00	01	02	03	04	05	06	07	08	09	10	11	12
00	00	00	00	00	00	00	00	00	00	00	00	00	00
01	00	01	02	03	04	05	06	07	08	09	10	11	12
02	00	02	04	06	08	10	12	14	16	18	20	22	24
03	00	03	06	09	12	15	18	21	24	27	30	33	36
04	00	04	08	12	16	20	24	28	32	36	42	44	48
05	00	05	10	15	20	25	30	35	40	45	50	55	60
06	00	06	12	18	24	30	36	42	48	54	60	66	72
07	00	07	14	21	28	35	42	49	56	63	70	77	84
08	00	08	16	24	32	40	48	56	64	72	80	88	96
09	00	09	18	27	36	45	54	63	72	81	90	99	108
10	00	10	20	30	40	50	60	70	80	90	100	110	120
11	00	11	22	33	44	55	66	77	88	99	110	121	132
12	00	12	24	36	48	60	72	84	96	108	120	132	144